Memristor: From Basics to Deployment

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Abstract

Until now three fundamental passive elements are used to design electronic circuits: resistor, capacitor, and inductors. The fourth fundamental passive element called "memristor" is recently fabricated even though invented several decades ago. The element is named memristor as it combines the behavior of a memory and a resistor. Memristor is a two-terminal element whose resistance depends on the magnitude and direction, and duration of the applied voltage. Memristor remembers its most recent memristance when voltage was turned off and until the next time voltage is turn on and can provide dynamical-negative resistance. It thus has the promising characteristics to potentially revolutionize nanoelectronics. It can find applications in analog and digital circuits which are part of everyday use systems such as sensors and mobile phones. This article discusses different aspects of memristor including basic characteristics, models, fabrications, and circuits designs to provide a complete picture of the state-of-the art.

1 Memristor: What is it?

Three fundamental passive elements such as resistor, capacitor, and inductor are currently used to build electronic circuits. The fourth fundamental element called **memristor** has recently emerged. The memristor was originally proposed in 1971 [1], however remained largely a theoretical concept until the demonstration of actual fabricated devices exhibiting the characteristics of a memristor by HP labs in 2008 [2, 3]. The new two-terminal passive element is named memristor as it combines the behavior of a memory and a resistor (i.e. **mem**ory+**res**i**stor**). One of the basic properties, resistance, of a memristor depends on the magnitude, direction, and duration of the voltage applied across its terminals. Memristor remembers its most recent resistance value when applied voltage was turned off and until the next time

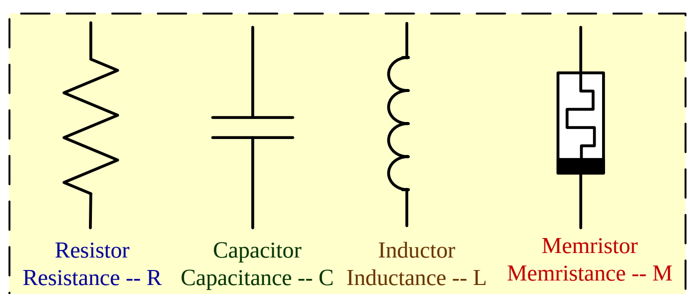


Figure 1: Memristor: The 4th Fundamental Element.

when applied voltage is turn on. Memristor has several interesting properties including pinched hysteresis and dynamical-negative resistance that can have significant impact on nanoelectronics. All four fundamental elements along with the memristor are presented in Fig. 1 for a comparative perspective.

The fundamental-electrical elements need to be discussed in the context of the fundamental-electrical variables, voltage (v), current (i), charge (q), and flux (ϕ) for clear understanding. The relations of fundamental elements and variables are presented in Fig. 2. Memristor is characterized by its memristance (M). This is describe by the charge-dependent rate of change of flux with charge as follows: $M(q) = (\mathrm{d}\Phi_m/\mathrm{d}q)$ [4]. This property is similar to the the fundamental element resistor which is characterized by its resistance (R). The other fundamental elements, inductor has inductance (L) and capacitor has capacitance (C) as their basic properties. It may be noted that the memristance is like a variable resistance. A battery can be considered to have memristance. However, the battery is an energy source and an active element, whereas the memristor is a passive element.

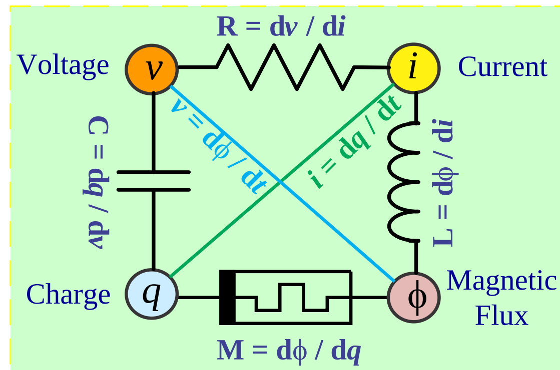


Figure 2: Memristor directly relates magnetic flux and charge; the missing connection among the 4 variables, v, i, q, and ϕ.

Memristors can be of various types depending on how they are built. A brief overview of different memristors is presented in Fig. 3. In addition there are systems which as a whole exhibit properties of memristors and hance are called “memristive systems”. Titanium dioxide (TiO_2) thinfilm memristors are the first ones to be built and widely explored for modeling and design. Polymeric or ionic memristors utilize dynamic doping of polymer and inorganic dielectric-type (some form of dioxide) materials. In this type of memristors, solid-state ionics (either cationic or anionic) move throughout the structure as the charge carriers. The resonant-tunneling diode memristors use specially doped quantum-well diodes. The manganite memristors use a substrate of bilayer oxide films based on manganite as opposed to titanium

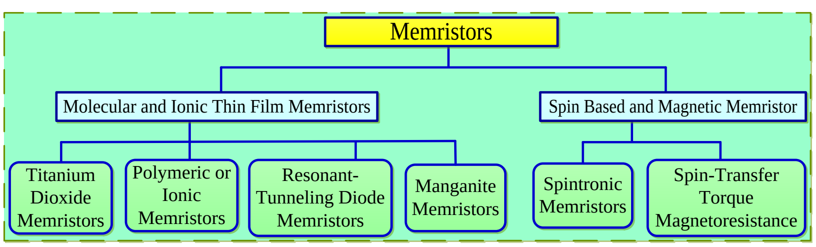


Figure 3: Different types of memristors.

dioxide. In spintronic memristors, the direction of spin of electrons change the magnetization state of the device which consequently changes its resistance. In the spin-transfer torque memristors, the relative magnetization alignment of the two electrodes affect the magnetic state of a magnetic tunnel junction which in turn changes its resistance.

2 Can Memristor be Fabricated to be Useful for Real-Life Chips?

Any circuit element, no matter how promising may look in theory can not be used to build any practical circuits or systems unless it can be fabricated. The lack of manufacturability didn't attract the attention of researchers from memristor even though it was introduced in 1971. However, now few industrial house (like HP labs) and academic clean rooms claim to build memristors. Memristors need to be manufactured with high yield to be used in chips and keep electronics cost affordable. As a specific example, the structure of a fabricated titanium dioxide thinfilm memristor is shown in Fig. 4 [5].

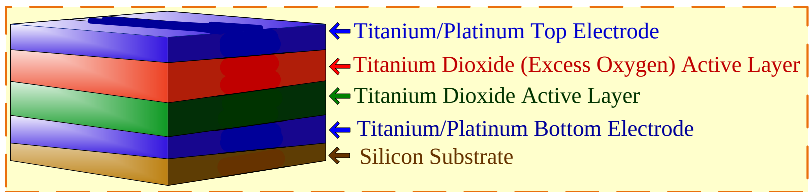


Figure 4: A TiO_2/TiO_{2+x} active layer thinfilm memristor.

The titanium dioxide thinfilm memristor consists of the following distinct layers: (1) Layer – 1: the bottom titanium/platinum (Ti/Pt) bilayer electrode. (2) Layer – 2: active titanium dioxide (TiO_2) layer. (3) Layer – 3: active titanium dioxide with excess oxygen (TiO_{2+x}) layer. (4) Layer – 4: the top titanium/platinum (Ti/Pt) bilayer electrode. The top and bottom Ti/Pt electrodes are metal connections. The TiO_{2+x} with excess oxygen provides charge carriers when voltage is applied across the top/bottom electrodes. The charge carriers then flow towards the active TiO_2 layer; thus, changing the resistance of TiO_2 layer and that of the memristor (this decreases the resistance). On the other hand, if the current direction is reversed through the memristor electrodes then the excess charge carriers from the TiO_2 layer moves towards the TiO_{2+x} layer (this increases the resistance).

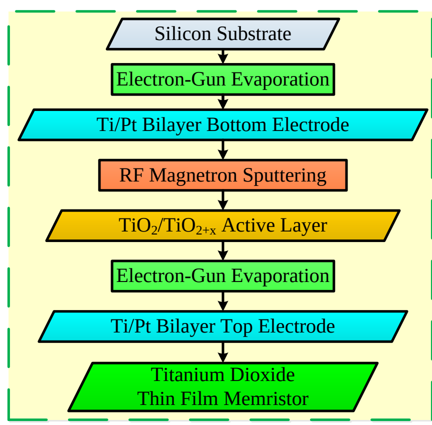


Figure 5: Memristor fabrication.

The above memristor was built using standard photolithography processes on a on silicon substrate. The fabrication steps are depicted in Fig. 5. A few nanometers thick titanium and platinum (Ti/Pt) bilayer is deposited on the silicon substrate by electron beam evaporation. This is followed by deposition of a layer of titanium dioxide (TiO_2) by radio frequency magnetron sputtering at room temperature. The titanium dioxide with excess oxygen (TiO_{2+x}) layer is then formed using the same process. The TiO_{2+x} layer is made non-stoichiometric with addition of excess oxygen atoms by flowing oxygen gas during the deposition. These titanium dioxide layers are the active layers of the memristor device. An additional layer of Ti/Pt bilayer is deposited for the top electrode; thus resulting in a complete memristor.

3 Are the Memristor's Models Available for Design Engineers?

Recently there has been an increased interest in research on memristors due to the demonstration of memristor manufacturing as well as their potential applications. Research is in full swing to use memristors in computer memory, analog circuits, sensors, and digital logic. Memristor models need to be made available for the design engineers to use the memristor as a circuit element during design exploration. However, the associate models for design and exploration usage by designers are available in limited forms only. Three types of models are available MATLAB, Verilog-A, and SPICE; but these are in primitive forms. MATLAB and Verilog-A models can be used for high-level abstracted simulations only; but not in real circuit design [6]. A recent SPICE model is available for circuit-level simulation [7]. However, a memristor layout library is not available for physical design of a chip.

As is the case of any new technology, it is important to learn how the memristor behaves to external stimulus in terms of voltage and current. It is stated that the memristance of a memristor depends on the amount and direction of charge that recently flowed through. It retains the last memristance when charge flow is stopped. At the same time when the charge flow resumes again the memristance of memristor changes from its last value. These characteristics memristor needs discussion for clear understanding of the device.

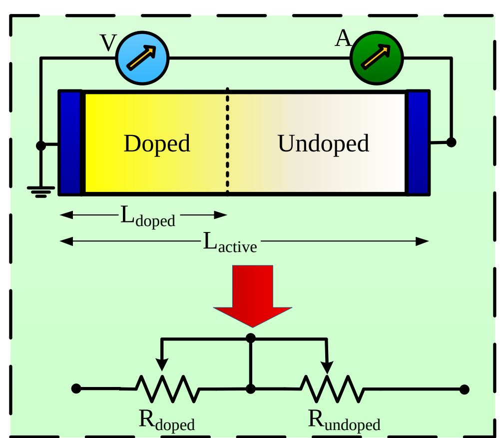


Figure 6: Memristor biasing.

The schematic diagram of a memristor for the purpose of characteristic analysis is presented in Fig. 6. The figure depicts that the bias voltage causes a drift of the dopants and electrically divides the memristor to doped and undoped regions. A small doped-region (which in means large undoped-region) provides higher resistance and a large doped-region (which means small undoped-region) provides lower resistance. Thus, transforming resistance between low and high values. For the purpose of analysis, the following parameters are assumed for the memristor: (1) L_{active} – the total active length of the memristor. This remains fixed once a memristor is manufactured. (2) $l_{doped}(t)$ – the doped active length of the memristor. This changes with the voltage applied across the two terminals. (3) R_{doped} – resistance of the doped layer of length L_{active}. This is equivalent to ON state resistance of the memristor R_{ON} as used in some literature. (4) $R_{undoped}$ – resistance of the undoped layer of length L_{active}. This is equivalent to OFF state resistance of the memristor R_{OFF} as used in some literature. (5) $v(t)$ – the applied biasing voltage across the memristor. (6) $q(t)$ – the resulting charge in the memristor. (7) $i(t)$ – the resulting current through the memristor. (8) μ – the average carrier mobility.

Applying Kirchhoff's voltage law (KVL) on the equivalent circuit (Fig. 6) of the memristor the following expression is obtained:

$$v(t) = \left(R_{doped} \left(\frac{l_{doped}(t)}{L_{active}} \right) + R_{undoped} \left(1 - \frac{l_{doped}(t)}{L_{active}} \right) \right) i(t). \tag{1}$$

For linear drifting with uniform field, the doped active length $l_{doped}(t)$ is the product of carrier velocity and carrier drifting time. The carrier velocity is obtained as product of μ and applied electric field of the following form: $(= v(t)/L_{active})$. The drifting time can be calculated from ratio of charge and current of the following form: $(q(t)/i(t))$. Thus, the following expression can be deduced:

$$l_{doped}(t) = \mu \left(\frac{v(t)}{L_{active}} \right) \left(\frac{q(t)}{i(t)} \right) = \mu q(t) \left(\frac{R_{doped}}{L_{active}} \right). \tag{2}$$

Assuming that R_{doped} is very small compared to $R_{undoped}$, the following is derived from Eqn. 1:

$$\frac{v(t)}{i(t)} = (R_{doped} - R_{undoped}) \left(\frac{l_{doped}(t)}{L_{active}} \right) + R_{undoped}. \tag{3}$$

By substituting Eqn. 2 in Eqn. 3, the following expression is obtained for the memristance (M):

$$M(q) = \frac{v(t)}{i(t)} = \left(1 - \left(\frac{\mu R_{doped}}{L_{active}^2} \right) q(t) \right) R_{undoped}. \tag{4}$$

The above equations form the basis of memristor characterization. Various different characteristics of the memristor for a sinusoidal input voltage of amplitude 1 volt and period 0.5 sec (i.e. $v(t) = \sin 4\pi t$) is presented in Fig. 7. These are generated using publicly available MATLAB models [6]. Similar characteristics can also be obtained using Verilog-A and SPICE models. Of course, different set of tools are needed for Verilog-A and SPICE. In the later case electronic-design automation (EDA) or computer-aided design (CAD) tools supporting behavioral Verilog-A and SPICE models with built-in macromodel (as compact models are not available) will be needed.

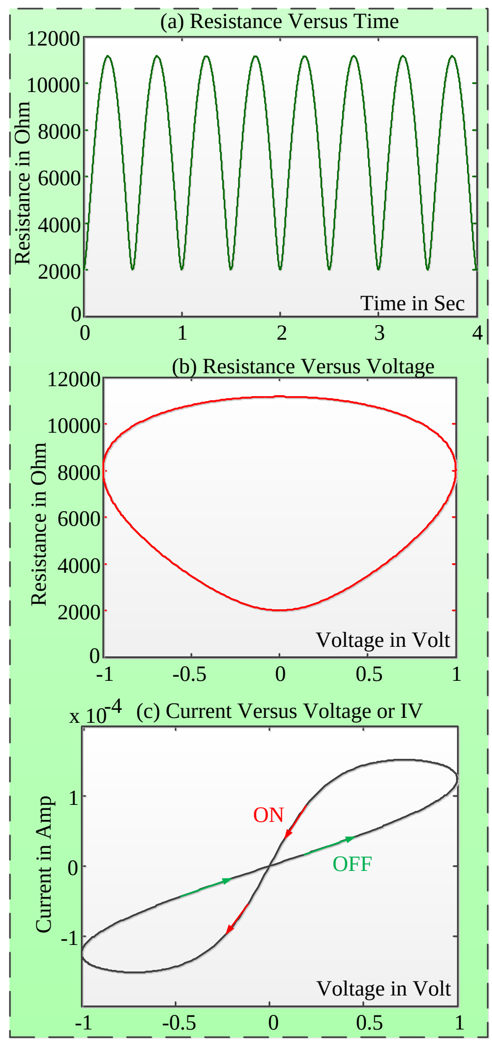


Figure 7: Characteristics of a memristor.

The resistance versus time characteristic of the memristor is presented in Fig. 7(a). The instantaneous resistance is in the range $[R_{ON}, R_{OFF}]$. The resistance values depend on the applied voltage. For a sine-wave voltage with period T, the memristance has its extreme (maximum or minimum) values at the following time instances: $t = (2n+1)T/2$ [6]. The resistance versus voltage characteristic is presented in Fig. 7(b). Initially the voltage across the memristor is 0 Volt and the current is 0 Amp, and a resistance of R_i [6]. The memristance value also depends on the sign of $v(t)$; in other words, resistance $[R_i, R_{OFF})$ for $v(t) < 0$ and $(R_{ON}, R_i]$ for $v(t) > 0$. This is due to the fact that the current follows voltage while resistance keeps increasing as long as voltage is positive. When voltage fall back to 0 Volt, resistance is maximum resistance R_{OFF}. The shape of memristance versus applied voltage curve is a $sin^2()$ function [6]. The current versus voltage (I-V) characteristic of the memristor is presented in Fig. 7(c) which shows its pinched hysteresis effect. The change in slope of the I-V characteristic demonstrates a switching between different resistance states; where the resistance is positive when the applied voltage increases and the negative when decreases. The symmetrical voltage bias results in double-loop I-V hysteresis which can collapse to a straight line for high frequencies.

From experiments as well as theoretical derivations of Eqn. 4 the concept of “dynamical negative” differential resistance exhibited by the memristor can be explained. From Eqn. 3 and Eqn. 4 it is evident that when the doped length (l_{doped}) is bound by $[0, L_{active}]$, the memristance (M) is bound by $[R_{doped}, R_{undoped}]$. When (l_{doped}) reaches either 0 or L_{active}, it remains constant until the applied voltage changes it polarity. At the upper boundary (i. e. $l_{doped} = L_{active}$), when the derivative of the voltage is negative, the memristor can exhibit a “dynamical negative” differential resistance. As an example, for a sin waveform the derivative of the voltage (i. e. $dv(t)/dt$) is negative during the phase 90°-180°. The dynamic negative effect is due to the charge-dependent change in the memristor resistance [3]. The negative resistance of memristors has attracted significant attentions from analog design engineers as in essence it can be used for efficient design of high-speed nanoelectronic oscillator circuits.

4 Emulators for the Memristor

At this point of memristor-technology it is effectively impossible to implement actual memristor-based circuits and systems to study their efficacy and functionality due to lack of real memristors to the designers. The available limited options need specialized memristor processes which do not integrate with traditional VLSI and are out of reach for most researchers. An approach investigated is to build discrete-component emulators of the memristor for its study without using the actual models for research advancement [8].

The block diagram of an emulator is presented in Fig. 8 [8]. It consists of three distinct components, Analog-to-Digital Converter (ADC), digital potentiometer, and microcontroller. In this memristor-emulator, the potentiometer's resistance changes continuously. The microcontroller received inputs from the ADC and updates the potentiometer. The preprogrammed equations of the memristor dictates the updating.

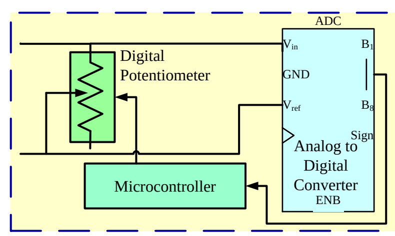
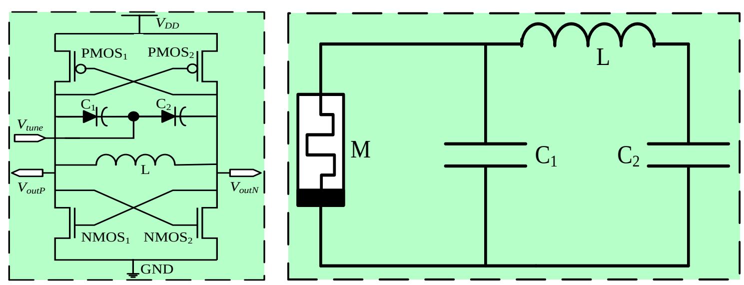


Figure 8: A memristor emulator.

The generalization to any class of two-terminal devices is possible by expanding the original memristor idea. The logical extension of this perception is the memory possessing capacitor and inductor which are known as memcapacitor and meminductor, respectively. The new three elements, memristor, memcapacitor, and meminductor, are together known as “memdevices”. Emulators for memcapacitor and meminductor are also implemented by using memristor-emulator in conjunction with operational-amplifiers (OP-AMPs) in various feedback paths [8].

5 Memristors in Analog Nanoelectronics

The inductor-capacitor tank based voltage-controlled oscillator (LC-VCO) presented in Fig. 9(a) is typically used as an electronic oscillator to control the frequency of the phase-locked loop (PLL). A PLL is the heart of every synchronous circuit or system needing a global clock [9]. The LC-VCO circuit is typically controlled by applying a DC input voltage through a loop filter. The LC-VCO produces cleaner output; however it occupies a significant area of the chip and sizeable portion of the power budget. An alternative of LC-VCO, the ring oscillator has high phase noise, is highly sensitive to disturbance, and has poor stability at high oscillation frequencies.



(a) Traditional LC-VCO. (b) Memristor-VCO.

Figure 9: Circuits of analog oscillators.

As a mitigation of the issues of the existing oscillators, a memristor-based VCO are explored. The schematic diagram of the memristor-VCO is presented in Fig. 9(b) [10]. In this design the memristor works with LC-tank for sustained oscillations. The memristor can be used for oscillator design as it provides negative resistance. In the negative resistance region, the memristor in essence behaves as an active device (like transistor) and hence can maintain sustainable oscillations. Thus, by replacing the transistors from the traditional LC-VCO circuit designs (in Fig. 9(a)) memristor-based VCO circuit designs (in Fig. 9(b)) are explored by the designers.

6 Memristors in Digital Nanoelectronics

In principle, a memristor can be used as a switch if an applied voltage will make severe change in the memristance. In such a memristor switch the time and energy that must be spent to achieve a targeted change in memristance are key quality factors. For a memristor to switch from R_{ON} to R_{OFF} in time T_{ON} to T_{OFF} the change in charge is quantified as $Q = Q_{ON} - Q_{OFF}$. The energy consumed for such a switching is quantified as $E_{switch} = VQ$, where $v(t) = V$ is a constant supply voltage. Thus, there will be switching energy dissipation as in the case of CMOS.

Most of the existing research on memristor is focused on cross-bar based memory design. Memristive Programmable Logic Arrays (PLAs) are also being explored for efficient reconfiguration in field-programmable gate-array (FPGA) implementation. For any technology to be useful in the design of main stream digital circuits and systems it needs to be functionally complete. In terms of Boolean logic, the basic gates like AND, OR, NOT needs to be designed for AND-OR-NOT implementations of digital functions. At the same time design of universal gates NAND/NOR can enable NAND-based or NOR-based digital design. It is observed that memristors can realize "implication logic" instead of Boolean logic [11]. Consequently, the implication logic which is functionally complete can be used to realize any Boolean functions.

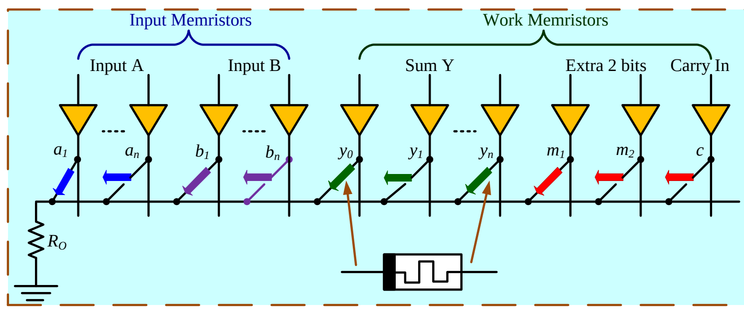


Figure 10: A memristor-based n-bit adder.

The block diagram of a n-bit full adder realized using memristors is presented in Fig. 10 [11]. It consists of two distinct set of memristors. The "input memristors" corresponds to input variables A and B. For a n-bit A and n-bit B there is a need of $2n$ input memristors. The "work memristors" corresponds to number of sum bits, two addition bits, and a carry bit. The additional two bits m_1 and m_2 are used during computation. Thus, $n + 3$ work memristors are used in this particular adder. In summary, a total of $3n + 3$ memristors are needed and these are connected with $3n + 3$ tristate buffers. For the computation purposes, resistance R_O needs to be much larger than any memristance of the input or work memristors and $R_{ON} < R_O < R_{OFF}$. The two states of a memristor are as follows: high memristance (or OFF state) is logic "0" (or FALSE) and a low memristance (or ON state) is logic "1" (or TRUE). Each of the memristor is connected with a tristate buffer; thus there are $3n+3$ of these. A memristor is logic 1 (i.e. "set") when a negative voltage V_{SET} is applied to its tristate buffer. Oppositely, a memristor is logic 0 (i.e. "clear") when a negative voltage V_{CLEAR} is applied to its tristate buffer. Another negative voltage V_{COND} (which is smaller than V_{SET}) is applied along with V_{SET} to execute a conditional switching operation; however, it does not change the state of a memristor like V_{SET}.

7 Conclusions

The new circuit element memristor was proposed in 1971. However, it was first manufactured in 2008. Since then multi-front research has been undertaken for using the memristor in practical circuits and systems.

Memristors which are presently being scaled up to 3nm has potential beyond nanoscale CMOS which faces lots of challenges beyond 18nm. The memristor models which are needed by the design engineers are still not mature to be used for accurate SPICE simulations. There are not layout libraries available for physical design of a the memristor or memristor-based circuits. MATLAB, Verliog-A, and primitive SPICE models are available for simulation of memristor-based circuits and systems. Research is in full swing for better SPICE models. Use of memristor for analog circuits such as voltage controlled and chaotic oscillators are explored at the circuit level. Cross-bar memory design using memristors is well researched. Digital design using memristor will need lots of research effort as Boolean logic can't be directly implemented. It is stated that the solid-state memristors combined with crossbar latches could potentially replace transistors in nanoelectronic circuits and build then in smaller area. It can be used to make high-density non-volatile solid-state resistive memory. The memristor along with the memcapacitor and the meminductor can potentially provide ultra-low power circuits and sensors.

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